ANALYTICAL METHOD FOR SOLVING NONLINEAR MULTILAYER ANISOTROPIC PLATE THEORY PROBLEMS

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By analyzing the known nonlinear solution for an elastic uniform circular plate with fixed edges, in 1955 Berger [1] suggested that the second invariant of the stress tensor for the center of the surface does not have any significant effect on the amount of deflection and it is permissible to ignore it in expressions for plate deformation energy. Subsequent alternative development of starting relationships of the problem led to differential equations, one of which is linear in relation to deflection. The elegant form of the equations, the possibility of applying to them known methods of solving linear boundary problems, has drawn the attention to many scientists, especially overseas, which is possible to judge from the review in [2] (see also [3-5]).

1. We construct equations of the Berger type for multilayer anisotropic plates and use them in order to solve nonlinear statics problems. We consider a multilayer anisotropic plate of constant thickness h. The datum surface Σ is referred to a set of curvilinear coordinates α^i. It is noted that in this section all indices with the exception of k = 1, 2, ..., N (N is the number of layers) take the values 1 and 2.

Displacements and deformations in the plate are determined by the equations [3]

$$u_i^{(k)} = u_i + z\theta_i + g(z)\psi_i,\ u_3^{(k)} = w,\ \theta_i = -\nabla_i w,\ g(z) = \int_0^z f_{(0)}(t)\,dt,$$

$$\varepsilon_{ij}^{(k)} = e_{ij} + z\varkappa_{ij} + g(z)\psi_{ij},\ \ \varepsilon_{i3}^{(k)} = f_{(0)}(z)\gamma_{i3},\ e_{ij} = (\nabla_i u_j + \nabla_j u_i + \nabla_i w\cdot\nabla_j w)/2,$$

$$\varkappa_{ij} = (\nabla_i\theta_j + \nabla_j\theta_i)/2,\ \psi_{ij} = (\nabla_i\psi_j + \nabla_j\psi_i)/2$$

[$f_{(0)}(z)$ is an a priori prescribed function of transverse coordinate z characterizing the rule for distribution of transverse shears through the package thickness).

Before starting to derive Berger type equations we turn attention to an important situation. Berger himself and the overwhelming majority of his successors by caring little about substantiating the hypothesis depart from the principle of possible displacements and from where equilibrium equations are derived in relation to displacements. In this way conformity was not established between force and kinematic characteristics for the plate, which often led to an incorrect understanding of the operating features of a structure and errors in formulating boundary conditions. These contradictions may be avoided if a mixed alternative principle is used of the form in [6]. Functional J_7 from [6] is presented in the form

$$\begin{aligned} J_7 = \iint_\Sigma \Big\{ W - T^{\alpha\beta}\Big[e_{\alpha\beta} - \frac{1}{2}(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha + \nabla_\alpha w\cdot\nabla_\beta w)\Big] - \\ - M^{\alpha\beta}\Big[\varkappa_{\alpha\beta} - \frac{1}{2}(\nabla_\alpha\theta_\beta + \nabla_\beta\theta_\alpha)\Big] - L^{\alpha\beta}\Big[\psi_{\alpha\beta} - \frac{1}{2}(\nabla_\alpha\psi_\beta + \nabla_\beta\psi_\alpha)\Big] - \\ - Q_o^\alpha(\gamma_{\alpha3} - \psi_\alpha)\Big\}\sqrt{a}\,d\alpha^1 d\alpha^2 - \iint_\Sigma \Big\{(p_+^\alpha - p_-^\alpha)u_\alpha + (\delta_{(N)}p_+^\alpha - \delta_{(0)}p_-^\alpha)\theta_\alpha + \\ + \big[g(\delta_{(N)})p_+^\alpha - g(\delta_{(0)})p_-^\alpha\big]\psi_\alpha + (q_+ - q_-)w\Big\}\sqrt{a}\,d\alpha^1 d\alpha^2 - \\ - \oint_\Gamma (T_{\nu\nu}^* u_\nu + T_{\nu t}^* u_t + M_{\nu\nu}^*\theta_\nu + M_{\nu t}^*\theta_t + L_{\nu\nu}^*\psi_\nu + L_{\nu t}^*\psi_t + Q_{\nu 3}^* w)\,ds_t. \end{aligned} \tag{1.1}$$

Here p_+^i, p_-^i, q_+, q_- are surface loads; $\delta_{(0)}$ and $\delta_{(N)}$ are distances from surface Σ to outer surfaces Σ_-, Σ_+; T^{ij}, Q_o^i, M^{ij}, L^{ij} are specific forces and moments determined by equations

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in [3]; $T_{\nu\nu}^*$, ..., $L_{\nu t}^*$, u_ν, ..., ψ_t are physical components of the corresponding tensors and vectors in a coordinate system s_t, s_ν, connected with boundary contour Γ; a is discriminant for the metric tensor surface Σ; W is specific deformation energy for the plate:

$$\begin{gathered}
W = (1/2)\, A^{\alpha\beta\gamma\omega} e_{\alpha\beta} e_{\gamma\omega} + B^{\alpha\beta\gamma\omega} e_{\alpha\beta} \varkappa_{\gamma\omega} + D^{\alpha\beta\gamma\omega} e_{\alpha\beta} \psi_{\gamma\omega} + \\
+ (1/2)\, C^{\alpha\beta\gamma\omega} \varkappa_{\alpha\beta} \varkappa_{\gamma\omega} + F^{\alpha\beta\gamma\omega} \varkappa_{\alpha\beta} \psi_{\gamma\omega} + (1/2)\, G^{\alpha\beta\gamma\omega} \psi_{\alpha\beta} \psi_{\gamma\omega} + (1/2)\, P^{\alpha\beta} \gamma_{\alpha 3} \gamma_{\beta 3}, \\
A^{\alpha\beta\gamma\omega} = \sum_{k=1}^{N} \int\limits_{\delta_{(k-1)}}^{\delta_{(k)}} b_{(k)}^{\alpha\beta\gamma\omega} dz, \quad B^{\alpha\beta\gamma\omega} = \sum_{k=1}^{N} \int\limits_{\delta_{(k-1)}}^{\delta_{(k)}} b_{(k)}^{\alpha\beta\gamma\omega} z\, dz, \\
C^{\alpha\beta\gamma\omega} = \sum_{k=1}^{N} \int\limits_{\delta_{(k-1)}}^{\delta_{(k)}} b_{(k)}^{\alpha\beta\gamma\omega} z^2 dz, \quad D^{\alpha\beta\gamma\omega} = \sum_{k=1}^{N} \int\limits_{\delta_{(k-1)}}^{\delta_{(k)}} b_{(k)}^{\alpha\beta\gamma\omega} g(z)\, dz, \\
F^{\alpha\beta\gamma\omega} = \sum_{k=1}^{N} \int\limits_{\delta_{(k-1)}}^{\delta_{(k)}} b_{(k)}^{\alpha\beta\gamma\omega} z g(z)\, dz, \quad G^{\alpha\beta\gamma\omega} = \sum_{k=1}^{N} \int\limits_{\delta_{(k-1)}}^{\delta_{(k)}} b_{(k)}^{\alpha\beta\gamma\omega} g^2(z)\, dz, \\
P^{\alpha\beta} = \sum_{k=1}^{N} \int\limits_{\delta_{(k-1)}}^{\delta_{(k)}} b_{(k)}^{\alpha 3\beta 3} f_{(0)}^2(z)\, dz
\end{gathered} \tag{1.2}$$

[$b_{(k)}^{\alpha\beta\gamma\omega}$ are tangential stiffnesses of the k-th layer].

We introduce generalized deformations e_{ij}^0 and generalized displacements u_i^0 relating to them:

$$e_{ij} = e_{ij}^{o} - z_1^{o} \varkappa_{ij} - z_2^{o} \psi_{ij}, \quad u_i = u_i^{o} - z_1^{o} \theta_i - z_2^{o} \psi_i \qquad (z_i^{o} = \text{const}). \tag{1.3}$$

Then we refer specific moments to a certain surface standing from the original surface by distance z_1^0, and generalized specific moments are referred to a surface standing at a distance of z_2^0 from the original surface:

$$M_o^{ij} = M^{ij} - z_1^{o} T^{ij}, \quad L_o^{ij} = L^{ij} - z_2^{o} T^{ij}. \tag{1.4}$$

Similar transformations should also be carried out with contour moments $M_{\nu\nu}^*$, $M_{\nu t}^*$, $L_{\nu\nu}^*$, $L_{\nu t}^*$.

By substituting generalized deformations and displacements from (1.3) in (1.1) and considering (1.2) and (1.4) we obtain

$$\begin{gathered}
J_7 = \iint\limits_{\Sigma} \Big\{ W - T^{\alpha\beta} \Big[e_{\alpha\beta}^{o} - \tfrac{1}{2} \left(\nabla_\alpha u_\beta^{o} + \nabla_\beta u_\alpha^{o} + \nabla_\alpha w \cdot \nabla_\beta w \right) \Big] - \\
- M_o^{\alpha\beta} \Big[\varkappa_{\alpha\beta} - \tfrac{1}{2} \left(\nabla_\alpha \theta_\beta + \nabla_\beta \theta_\alpha \right) \Big] - L_o^{\alpha\beta} \Big[\psi_{\alpha\beta} - \tfrac{1}{2} \left(\nabla_\alpha \psi_\beta + \nabla_\beta \psi_\alpha \right) \Big] - \\
- Q_o^{\alpha} (\gamma_{\alpha 3} - \psi_\alpha) - (p_+^{\alpha} - p_-^{\alpha})\, u_\alpha^{o} - \left[(\delta_{(N)} - z_1^{o})\, p_+^{\alpha} - (\delta_{(0)} - z_1^{o})\, p_-^{\alpha} \right] \theta_\alpha - \\
- \left[(g(\delta_{(N)}) - z_2^{o})\, p_+^{\alpha} - (g(\delta_{(0)}) - z_2^{o})\, p_-^{\alpha} \right] \psi_\alpha - (q_+ - q_-)\, w \Big\} \sqrt{a}\, d\alpha^1 d\alpha^2 - \\
- \oint\limits_{\Gamma} \left(T_{\nu\nu}^{*} u_\nu^{o} + T_{\nu t}^{*} u_t^{o} + M_{\nu\nu}^{o*} \theta_\nu + M_{\nu t}^{o*} \theta_t + L_{\nu\nu}^{o*} \psi_\nu + L_{\nu t}^{o*} \psi_t + Q_{\nu 3}^{*} w \right) ds_t;
\end{gathered} \tag{1.5}$$

$$\begin{gathered}
W = \tfrac{1}{2} A^{\alpha\beta\gamma\omega} e_{\alpha\beta}^{o} e_{\gamma\omega}^{o} + B_o^{\alpha\beta\gamma\omega} e_{\alpha\beta}^{o} \varkappa_{\gamma\omega} + D_o^{\alpha\beta\gamma\omega} e_{\alpha\beta}^{o} \psi_{\gamma\omega} + \\
+ \tfrac{1}{2} \left(C_o^{\alpha\beta\gamma\omega} - z_1^{o} B_o^{\alpha\beta\gamma\omega} \right) \varkappa_{\alpha\beta} \varkappa_{\gamma\omega} + \left(F_o^{\alpha\beta\gamma\omega} - z_1^{o} D_o^{\alpha\beta\gamma\omega} \right) \varkappa_{\alpha\beta} \psi_{\gamma\omega} + \tfrac{1}{2} \left(G_o^{\alpha\beta\gamma\omega} - z_2^{o} D_o^{\alpha\beta\gamma\omega} \right) \psi_{\alpha\beta} \psi_{\gamma\omega} + \tfrac{1}{2} P^{\alpha\beta} \gamma_{\alpha 3} \gamma_{\beta 3},
\end{gathered} \tag{1.6}$$

where $B_o^{\alpha\beta\gamma\omega}$, $C_o^{\alpha\beta\gamma\omega}$, $D_o^{\alpha\beta\gamma\omega}$, $F_o^{\alpha\beta\gamma\omega}$, $G_o^{\alpha\beta\gamma\omega}$ are cited stiffnesses for the plate:

$$\begin{gathered}
B_o^{\alpha\beta\gamma\omega} = B^{\alpha\beta\gamma\omega} - z_1^{o} A^{\alpha\beta\gamma\omega}, \; C_o^{\alpha\beta\gamma\omega} = C^{\alpha\beta\gamma\omega} - z_1^{o} B^{\alpha\beta\gamma\omega}, \; D_o^{\alpha\beta\gamma\omega} = D^{\alpha\beta\gamma\omega} - \\
- z_2^{o} A^{\alpha\beta\gamma\omega}, \; F_o^{\alpha\beta\gamma\omega} = F^{\alpha\beta\gamma\omega} - z_2^{o} B^{\alpha\beta\gamma\omega}, \; G_o^{\alpha\beta\gamma\omega} = G^{\alpha\beta\gamma\omega} - z_2^{o} D^{\alpha\beta\gamma\omega}.
\end{gathered}$$

Presence in expressions for specific deformation energy of the plate (1.6) of second and third terms markedly limits the region for application of the Berger hypothesis in multilayer anisotropic plate theory since moments and generalized moments depend on elongations and shears for the datum surface, and in equations for tangential forces terms are present taking account of the effect of parameters for the change in curvature and transverse shears.

We simplify Eq. (1.6) by assuming that coefficients in cross terms equal zero:

$$B_o^{\alpha\beta\gamma\omega}=0,\quad D_o^{\alpha\beta\gamma\omega}=0. \qquad (1.7)$$

Equalities (1.7) may be treated as conditions for transforming calculation for a layered plate to a uniform plate, and they were first obtained in [7] for bimetallic structures. It is easy to be certain that in the general case of a multilayer anisotropic plate conditions for deriving (1.7) are identically only satisfied for a symmetrically assembled package of layers. For transversely isotropic plates (1.7) is also fulfilled for asymmetrical packages when Poisson's ratios for the layers are equal. Then by introducing the relative Poisson's ratio we obtain equations for parameters z_i^o from (1.3), (1.4): $z_1^o = hc_{13}/2$, $z_2^o = hc_{12}/2$, where c_{12} and c_{13} are dimensionless stiffness parameters [3].

We use the Berger hypothesis by approximately presenting (1.6) in the form

$$W\simeq\frac{1}{2}I_1^2+\frac{1}{2}C_o^{\alpha\beta\gamma\omega}\varkappa_{\alpha\beta}\varkappa_{\gamma\omega}+F_o^{\alpha\beta\gamma\omega}\varkappa_{\alpha\beta}\psi_{\gamma\omega}+\frac{1}{2}G_o^{\alpha\beta\gamma\omega}\psi_{\alpha\beta}\psi_{\gamma\omega}+\frac{1}{2}P^{\alpha\beta}\gamma_{\alpha3}\gamma_{\beta3}. \qquad (1.8)$$

The value

$$I_1=U^{\alpha\beta}e^o_{\alpha\beta} \qquad (1.9)$$

corresponds to the first invariant of the deformation tensor for the central plane of an anisotropic plate and for uniform orthotropic plates introduced for example in [8]. Here $U^{\alpha\beta}$ are contravariant components of a second rank symmetrical tensor: $U^{11}=\sqrt{A^{1111}}$, $U^{22}=\sqrt{A^{2222}}$, $U^{12}=\sqrt{2A^{1112}}+\sqrt{2A^{2212}}$.

We find the variation of functional J_7 from (1.5) taking account of (1.8) and (1.9), by submitting to variation u_i^o, ψ_i, w, e_{ij}^o, $\varkappa_{ij}$, ψ_{ij}, γ_{i3}, T^{ij}, M_o^{ij}, L_o^{ij}, Q_o^i:

$$\begin{aligned}\delta J_7=\iint_\Sigma\{&-(T^{\alpha\beta}-U^{\alpha\beta}I_1)\,\delta e^o_{\alpha\beta}-(M_o^{\alpha\beta}-C_o^{\alpha\beta\gamma\omega}\varkappa_{\gamma\omega}-F_o^{\alpha\beta\gamma\omega}\psi_{\gamma\omega})\,\delta\varkappa_{\alpha\beta}-\\&-(L_o^{\alpha\beta}-F_o^{\alpha\beta\gamma\omega}\varkappa_{\gamma\omega}-G_o^{\alpha\beta\gamma\omega}\psi_{\gamma\omega})\,\delta\psi_{\alpha\beta}-(Q_o^\alpha-P^{\alpha\beta}\gamma_{\beta3})\,\delta\gamma_{\alpha3}-\\&-\left[e^o_{\alpha\beta}-\frac{1}{2}(\nabla_\alpha u^o_\beta+\nabla_\beta u^o_\alpha+\nabla_\alpha w\cdot\nabla_\beta w)\right]\delta T^{\alpha\beta}-\left[\varkappa_{\alpha\beta}-\frac{1}{2}(\nabla_\alpha\theta_\beta+\nabla_\beta\theta_\alpha)\right]\delta M_o^{\alpha\beta}-\\&-\left[\psi_{\alpha\beta}-\frac{1}{2}(\nabla_\alpha\psi_\beta+\nabla_\beta\psi_\alpha)\right]\delta L_o^{\alpha\beta}-(\gamma_{\alpha3}-\psi_\alpha)\,\delta Q_o^\alpha-(\nabla_\alpha T^{\alpha\beta}+p_+^\beta-p_-^\beta)\,\delta u^o_\beta-\\&-[\nabla_\beta\nabla_\alpha M_o^{\alpha\beta}-\nabla_\beta(T^{\alpha\beta}\theta_\alpha)+(\delta_{(N)}-z_1^o)\nabla_\alpha p_+^\alpha-(\delta_{(0)}-z_1^o)\nabla_\alpha p_-^\alpha+q_+-q_-]\,\delta w-\\&-[\nabla_\alpha L_o^{\alpha\beta}-Q_o^\beta+(g(\delta_{(N)})-z_2^o)\,p_+^\beta-(g(\delta_{(0)})-z_2^o)\,p_-^\beta]\,\delta\psi_\beta\}\sqrt{a}\,d\alpha^1 d\alpha^2+\\&+\oint_\Gamma\Big\{(T_{\nu\nu}-T^*_{\nu\nu})\,\delta u^o_\nu+(T_{\nu t}-T^*_{\nu t})\,\delta u^o_t+(M^o_{\nu\nu}-M^{o*}_{\nu\nu})\,\delta\theta_\nu+\\&+(L^o_{\nu\nu}-L^{o*}_{\nu\nu})\,\delta\psi_\nu+(L^o_{\nu t}-L^{o*}_{\nu t})\,\delta\psi_t+\left[\frac{\partial M^o_{\nu t}}{\partial s_t}+\nu_\beta\nabla_\alpha M_o^{\alpha\beta}-T_{\nu\nu}\theta_\nu-T_{\nu t}\theta_t+\right.\\&\left.+(\delta_{(N)}-z_1^o)\,p_\nu^+-(\delta_{(0)}-z_1^o)\,p_\nu^--\frac{\partial M^{o*}_{\nu t}}{\partial s_t}-Q^*_{\nu3}\right]\delta w\Big\}\,ds_t.\end{aligned}$$

By substituting the equation obtained for δJ_7 in variation equation $\delta J_7 = 0$ we arrive at deformation expressions

$$e^o_{ij}=\frac{1}{2}(\nabla_i u^o_j+\nabla_j u^o_i+\nabla_i w\cdot\nabla_j w),\quad \gamma_{i3}=\psi_i,\quad \varkappa_{ij}=\frac{1}{2}(\nabla_i\theta_j+\nabla_j\theta_i),$$

$$\psi_{ij}=\frac{1}{2}(\nabla_i\psi_j+\nabla_j\psi_i);$$

elasticity relationships

$$T^{ij} = U^{ij} I_1, \quad M_o^{ij} = C_o^{ij\alpha\beta} \varkappa_{\alpha\beta} + F_o^{ij\alpha\beta} \psi_{\alpha\beta}, \quad Q_o^i = P^{i\alpha} \gamma_{\alpha 3}, \qquad (1.10)$$
$$L_o^{ij} = F_o^{ij\alpha\beta} \varkappa_{\alpha\beta} + G_o^{ij\alpha\beta} \psi_{\alpha\beta};$$

equilibrium equations

$$\nabla_\alpha T^{\alpha i} = p_-^i - p_+^i, \quad \nabla_\beta \nabla_\alpha M_o^{\alpha\beta} - \nabla_\beta (T^{\alpha\beta} \theta_\alpha) = q_- - q_+ + \qquad (1.11)$$
$$+ (\delta_{(0)} - z_1^o) \nabla_\alpha p_-^\alpha - (\delta_{(N)} - z_1^o) \nabla_\alpha p_+^\alpha, \quad \nabla_\alpha L_o^{\alpha i} - Q_o^i = [g(\delta_{(0)} - z_2^o)] p_-^i - [g(\delta_{(N)}) - z_2^o] p_+^i;$$

natural boundary conditions

$$(T_{\nu\nu} - T_{\nu\nu}^*) \delta u_\nu^o = 0, \quad (T_{\nu t} - T_{\nu t}^*) \delta u_t^o = 0, \quad (M_{\nu\nu}^o - M_{\nu\nu}^{o*}) \delta\theta_\nu = 0,$$
$$(L_{\nu\nu}^o - L_{\nu\nu}^{o*}) \delta\psi_\nu = 0, \quad (L_{\nu t}^o - L_{\nu t}^{o*}) \delta\psi_t = 0, \quad \left[\frac{\partial M_{\nu t}^o}{\partial s_t} + \nu_\beta \nabla_\alpha M_o^{\alpha\beta} - T_{\nu\nu}\theta_\nu - \right. \qquad (1.12)$$
$$\left. - T_{\nu t}\theta_t + (\delta_{(N)} - z_1^o) p_\nu^+ - (\delta_{(0)} - z_1^o) p_\nu^- - \frac{\partial M_{\nu t}^{o*}}{\partial s_t} - Q_{\nu 3}^* \right] \delta w = 0.$$

If in (1.11), (1.12) and (1.3), (1.4) we assume $z_i^o = 0$, then we obtain an equilibrium equation and boundary conditions for multilayer plate theory based on the generalized Timoshenko hypothesis [3]. This not by accident. In fact, tangential forces T^{ij} from (1.10) correspond to the Berger hypothesis adopted by substituting traditional forces in a multilayer plate. It is also noted that (1.10) and (1.11) in the form written coincide with relationships and equations of the three-layer anisotropic plate theory of the Berger type [9].

2. The procedure suggested for studying geometrically nonlinear problems is particularly simple and effective for multilayer transversely isotropic plates and it has a considerable physical clarity compared with the generally accepted Berger approach. For rectangular transversely isotropic plates equilibrium Eq. (1.11) may be written in the form

$$T_{1i,1} + T_{2i,2} = 0, \quad L_{1i,1}^o + L_{2i,2}^o = Q_{oi}, \qquad (2.1)$$
$$M_{11,11}^o + 2M_{12,12}^o + M_{22,22}^o + T_{11}w_{,11} + 2T_{12}w_{,12} + T_{22}w_{,22} = -q.$$

Starting from this point it is assumed that the datum surface of the plate is referred to a Cartesian coordinate system α_1, α_2, and T_{ij}, M_{ij}^o, L_{ij}^o, Q_{oi} are physical components of the corresponding tensors. Equilibrium Eqs. (2.1) in the form written coincide with equilibrium equations for three-layer transversely isotropic plates [10]. The results are also similar for boundary conditions and therefore they are not given here.

Equation (2.1) may be transformed if a method described in [3] is used. As a result we obtain differential equations concerning displacement functions χ and shear functions φ

$$\left(1 - \frac{\theta h^2}{\beta} \Delta\right) \Delta\Delta\chi - \alpha^2 \left(1 - \frac{h^2}{\beta}\Delta\right) \Delta\chi = \frac{q}{D}; \qquad (2.2)$$

$$\frac{1-\nu}{2} \frac{h^2}{\beta} \Delta\varphi = \varphi, \quad D = \frac{Eh^3}{12(1-\nu^2)} \eta_3 \qquad (2.3)$$

and an integrodifferential equation for determining Berger constants α^2

$$\frac{abh^2\eta_3}{6} \alpha^2 = \int_0^b \int_0^a [(w_{,1})^2 + (w_{,2})^2] \, d\alpha_1 \, d\alpha_2, \quad w = \left(1 - \frac{h^2}{\beta}\Delta\right) \chi, \qquad (2.4)$$

where ν is relative Poisson's ratio; E is averaged elasticity modulus; θ, β, η_3 are dimensionless stiffness parameters [3]; a, b are rectangular plate dimensions.

A virtue of differential Eqs. (2.2) and (2.3) is that they are linear and not connected with each other. In addition, (2.3) is a solution of the edge effect type. It makes it possible in studying certain special problems to assume approximately that $\varphi = 0$, and thus to reduce the general order of set of differential Eqs. (2.2), (2.3) from eight to six. In form of writing Eqs. (2.2)-(2.4) agrees with equations constructed in [10] for three-layer plates with a rigid transversely isotropic filler. This result is of considerable practical importance because it shows that in concept calculation of multilayer plates in no way differs from calculation of three-layer plates. Therefore, results obtained for three-layer plates of finite deflection, e.g., in [10], may be used directly in design calculations for multilayer plates.

For the case of cylindrical bending of a hinged plate subject to the action of uniform load q the solution of the problem may be written in the form

$$\chi = \frac{q}{\alpha^2 D}\left[\frac{\theta_2^2 \operatorname{ch}\theta_1 (a/2-\alpha_1)}{\theta_1^2(\theta_2^2-\theta_1^2)\operatorname{ch}(\theta_1 a/2)} + \frac{\theta_1^2 \operatorname{ch}\theta_2 (a/2-\alpha_1)}{\theta_2^2(\theta_1^2-\theta_2^2)\operatorname{ch}(\theta_2 a/2)} + \frac{1}{2}\alpha_1(a-\alpha_1) - \right.$$
$$\left. - \frac{1}{\theta_1^2} - \frac{1}{\theta_2^2}\right], \quad \theta_{1,2} = \left\{\frac{1+\alpha^2 h^2/\beta \mp [(1+\alpha^2 h^2/\beta)^2 - 4\theta\alpha^2 h^2/\beta]^{1/2}}{2\theta h^2/\beta}\right\}^{1/2}. \qquad (2.5)$$

By taking (2.5) shear parameter $\beta = \infty$ we arrive at the original solution of Bubnov [11] first obtained for a uniform isotropic plate in 1902.

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